

# Part Two Direct Proportion and Inverse Proportion



Let's discuss.

## **Direct proportion**

In the previous class we have learnt how to compare two numbers and write them in the form of a ratio.

**Example** Look at the picture below. We see divisions of a circle made by its diameters.



(A)



(B)



(C)



(D)

Do you see any relationship between the number of diameters and the number of divisions they give rise to?

In figure (A) **one** diameter makes

parts of the circle.

In figure (B) **two** diameters make

parts of the circle.

In figure (D) **four** diameters make

parts of the circle.

No. of diameters  $\frac{1}{1} = \frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$ . Here, the ratio of the number of diameters to the number of divisions remains constant.

Example The number of notebooks that the students of a Government School received is shown in the table below.

Children	15	12	10	5
Notebooks	90	72	60	30

$$\frac{\text{Number of children}}{\text{Number of notebooks}} = \frac{15}{90} = \frac{12}{72} = \frac{10}{60} = \frac{5}{30} = \frac{1}{6}$$

In other words, the ratio 1:6 remains the same or constant.

In the examples above, we see that when the number of diameters increases the number of divisions also increases. As the number of children decreases the number of notebooks also falls. The number of diameters and the number of divisions are in direct proportion as are the number of students and the number of notebooks.

Activity: \* Think: Are the amount of petrol filled in a motorcycle and the distance travelled by it, in direct proportion?

> \* Discuss: Can you give examples from science or everyday life, of quantities that vary in direct proportion?

**Example** If 10 pens cost 60 rupees, what is the cost of 13 such pens?

**Solution**: Let us suppose the cost of 13 pens is x rupees.

The number of pens and their cost vary in direct proportion. Let us express the ratios and obtain an equation.

$$\frac{10}{60} = \frac{13}{x}$$

 $\therefore$  10x = 780 (multiplying both sides by 60x)

$$\therefore x = 78$$

Cost of 13 pens is ₹ 78.

## **Practice Set 37**

- 1. If 7 kg onions cost 140 rupees, how much must we pay for 12 kg onions?
- 2. If 600 rupees buy 15 bunches of feed, how many will 1280 rupees buy?
- 3. For 9 cows, 13 kg 500 g of food supplement are required every day. In the same proportion, how much will be needed for 12 cows?
- 4. The cost of 12 quintals of soyabean is 36,000 rupees. How much will 8 quintals cost?
- 5. Two mobiles cost 16,000 rupees. How much money will be required to buy 13 such mobiles ?







Some volunteers have gathered to dig 90 pits for a tree plantation programme. One volunteer digs one pit in one day. If there are 15 volunteers, they will take  $\frac{90}{15} = 6$  days to dig the pits.

10 volunteers will take  $\frac{90}{10} = 9$  days.

Are the number of pits and the number of volunteers in direct proportion?

If the number of volunteers **decreases**, **more** days are required; and if the number of volunteers **increases**, **fewer** days are required for the job. However, the product of the number of days and number of volunteers remains constant. We say that these numbers are in **inverse** proportion.

• Suppose Sudha has to solve 48 problems in a problem set. If she solves 1 problem every day, she will need 48 days to complete the set. But, if she solves 8 problems every day, she will complete the set in  $\frac{48}{8} = 6$  days and if she solves 12 problems a day, she will need  $\frac{48}{12} = 4$  days. The number of problems solved in a day and the number of days needed are in inverse proportion. Their product is constant.

Thus, note that  $8 \times 6 = 12 \times 4 = 48 \times 1$ 

**Example** Fifteen workers take 8 hours to build a wall. How many hours will 12 workers need to build the same wall?

**Solution**: As the number of workers increases, the number of hours decreases. The number of workers and number of hours are in inverse proportion. The product of the number of workers and the number of hours needed to build the wall is constant. Let us use the variable *x* to solve this problem.

Suppose, 12 workers take x hours.  $12 \times x = 15 \times 8$ 15 workers take 8 hours.  $\therefore 12x = 120$ 12 workers take x hours.  $\therefore x = 10$ 

Thus, 12 workers will take 10 hours to build the wall.

**Example** A 40-page class magazine is to be written. If one student would require 80 days to write it, how many would 4 students require?

**Solution**: If more students help to do the same task, fewer days will be required. That is, the number of students and number of days are in inverse proportion. Suppose 4 students need *x* days.

Students	Days	$4x = 80 \times 1$	
1	80	$x = \frac{80}{4}$	
4	x	x = 20	∴ 4 students require

**Example** Students of a certain school went for a picnic to a farm by bus. Here are some of their experiences. Say whether the quantities in each are in direct or in inverse proportion.

• Each student paid 60 rupees for the expenses.

As there were 45 students, rupees were collected.

Had there been 50 students, rupees would have been collected.

The number of students and money collected are in ...... proportion.

• The sweets shop near the school gave 90 laddoos for the picnic.

If 45 students go for the picnic, each will get laddoos.

If 30 students go for the picnic, each will get laddoos.

The number of students and that of laddoos each one gets are in ...... proportion.

• The farm is 120 km away from the school.

The bus went to the farm at a speed of 40 km per hour and took hours.

On the return trip, the speed was 60 km per hour. Therefore, it took

On the return trip, the speed was 60 km per hour. Therefore, it took

The speed of the bus and the time it takes are in ..... proportion.

20 days.



The farmer picked 180 bors from his trees. He gave them equally to 45 students. Each student got

> Had there been 60 students, each would have got bors.

The number of students and the number of bors each one gets are in ....... proportion

#### **Practice Set 38**

- 1. Five workers take 12 days to weed a field. How many days would 6 workers take? How many would 15 take?
- 2. Mohanrao took 10 days to finish a book, reading 40 pages every day. How many pages must he read in a day to finish it in 8 days?
- 3. Mary cycles at 6 km per hour. How long will she take to reach her Aunt's house which is 12 km away? If she cycles at a speed of 4 km/hr, how long would she take?

bors.

4. The stock of grain in a government warehouse lasts 30 days for 4000 people. How many days will it last for 6000 people?



#### **Partnership**

When starting a business enterprise, money is required for an office, raw materials, etc. This amount is called the capital. Often, two or more people put in money for the capital. In other words, these people start a business by investing in the partnership. In a business partnership, all partners have a joint account in a bank. The profit made or the loss incurred is shared by the partners in proportion to the money each one has invested.

**Example** Jhelum and Atharva invested 2100 and 2800 rupees respectively and started a business. They made a profit of 3500 rupees. How should it be shared?

**Solution**: Let us find out the proportion of the investments.

 $2100:2800 = \frac{2100}{2800} = \frac{3}{4} = 3:4$ . : Proportion of investments is 3:4.

The profit must also be shared in the same proportion.

Let Jhelum's profit be 3x and that of Atharva, 4x. Then,

$$3x + 4x = 3500$$
 as total profit is 3500.

$$x = 3500$$
  $x = 500$ 

Jhelum's share = 3x = 1500 rupees and Atharva's share = 4x = 2000 rupees.

**Example** Chinmaya and Sam invested a total of 130000 rupees in a business in the proportion 3:2 respectively. What amount did each of them invest? If their total profit was 36000 rupees, what is the share of each?

**Solution**: The proportion of Chinmaya's and Sam's investment is 3:2.

The profit is shared in the same **proportion** as the investment, hence, proportion of profit is 3:2.



Let Chinmaya's investment be 3y and Sam's 2y. Let Chinmaya's profit be 3x and Sam's 2x.

$$3y + 2y = \text{Total investment.}$$
  

$$\therefore 5y = 130000$$

$$\frac{5y}{5} = \frac{130000}{5} \dots \text{ (dividing by 5)}$$

$$\therefore y = 26000$$

 $\therefore$  Chinmaya's investment = 3y

$$= 3 \times 26000$$
= 78,000

Sam's investment = 2y
= 2 \times 26000
= 52000 rupees

$$3x + 2x = \text{Total Profit}$$
 $5x = 36000$ 

$$\frac{5x}{5} = \frac{36000}{5} \cdot \cdot \cdot \cdot \text{ (dividing by 5)}$$

$$\therefore x = 7200$$

$$\therefore \text{ Chinmaya's profit} = 3x$$

$$= 3 \times 7200$$

$$= 21600$$
Sam's profit =  $2x$ 

$$=  $2 \times 7200$$$

$$= 14400 \text{ rupees}$$

**Example** Abdul, Sejal and Soham each gave Sayali 30 rupees, 70 rupees and 50 rupees respectively. Sayali put in 150 rupees and bought paper, colours, etc. Together they made greeting cards and sold them all. If they made a total profit of 420 rupees, what was each one's share in the profit?

The capital invested by all four was 300 rupees. Of this Sayali had invested Solution: 150 rupees, that is, half of the capital. The total profit was 420 rupees. So, Sayali's profit was half of that, i.e., 210 rupees. The remaining 210 was shared by Abdul, Sejal and Soham.

> Abdul, Sejal and Soham's investment is 30, 70 and 50 rupees. The proportion is 30:70:50 i.e. 3:7:5. Their share of the profit is altogether 210 rupees.

Let their individual profit be 3k, 7k, 5k. Then, 3k + 7k + 5k = 210

$$\therefore 15k = 210$$

$$\therefore k = 14$$

Abdul's profit =  $3k = 3 \times 14 = 42$  rupees.

Sejal's profit =  $7k = 7 \times 14 = 98$  rupees.

Soham's profit =  $5k = 5 \times 14 = 70$  rupees.

Saritaben, Ayesha and Meenakshi started a business by investing 2400, Example 5200 and 3400 rupees. They made a profit of 50%. How should they share it? If they reinvest all their profit by adding it to the capital, what will each one's share be in the following year?

Total capital = 2400 + 5200 + 3400 = 11000 rupees. **Solution**: Profit 50%

$$\therefore \text{ Total profit} = \frac{11000 \times 50}{100} = 5500$$

Profit will be shared in the same proportion as the investment.

We simplify the ratio of two numbers by dividing by a common factor. The same can be done for 3 or more numbers.

Assume that Saritaben's profit = 12p, Ayesha's profit = 26p, Meenakshi's profit = 17p.

$$\therefore 12p + 26p + 17p = 55p = 5500 \therefore p = \frac{5500}{55} = 100$$

:. Saritaben's profit =  $12 \times 100 = 1200$ , Ayesha's profit =  $26 \times 100 = 2600$  and Meenakshi's profit =  $17 \times 100 = 1700$ .

If they add their profit to the capital, their further investments will be:

Saritaben's capital = 
$$2400 + 1200 = ₹ 3600$$
  
Ayesha's capital =  $5200 + 2600 = ₹ 7800$   
Meenakshi's capital =  $3400 + 1700 = ₹ 5100$ 



In the above example, if Saritaben, Meenakshi and Ayesha all add their profit to the capital, find out the proportions of their shares in the capital during the following year.

# **Practice Set 39**

- 1. Suresh and Ramesh together invested 144000 rupees in the ratio 4:5 and bought a plot of land. After some years they sold it at a profit of 20%. What is the profit each of them got?
- 2. Virat and Samrat together invested 50000 and 120000 rupees to start a business. They suffered a loss of 20%. How much loss did each of them incur?
- 3. Shweta, Piyush and Nachiket together invested 80000 rupees and started a business of selling sheets and towels from Solapur. Shweta's share of the capital was 30000 rupees and Piyush's 12000. At the end of the year they had made a profit of 24%. What was

- Nachiket's investment and what was his share of the profit?
- 4. A and B shared a profit of 24500 rupees in the proportion 3:7. Each of them gave 2% of his share of the profit to the Soldiers' Welfare Fund. What was the actual amount given to the Fund by each of them?
- 5\*. Jaya, Seema, Nikhil and Neelesh put in altogether 360000 rupees to form a partnership, with their investments being in the proportion 3:4:7:6. What was Jaya's actual share in the capital? They made a profit of 12%. How much profit did Nikhil make?